THE REINSURANCE GAME—A TOOL FOR EDUCATION AND RESEARCH

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Business games, apparently invented only a few years ago, are already being used for training, education, and research in many countries.[1] These games can also be adapted for use in insurance education. At least one example, the McGuinness game, has already been published.[2] This article will outline another game which is being used in the reinsurance courses at the Copenhagen School of Economics and Business Administration. The construction and the rules of the game will be described in such a way that it can be used, and modified if desired, by other teachers.[3]

It should be stressed from the outset that the reinsurance game which will be described here, differs from all or most other games in some important respects, and that verdicts on the use and value of business games in general, therefore, may not hold for this game. First, the reinsurance game is purely stochastic while other games either are only partly probabilistic or are purely deterministic in the sense that there is a fixed functional relationship between the variables of the game. Second, the reinsurance game is a negotiation game while almost all other games known to the author are decision games.[4] In a decision game, each player, or group constituting a firm, makes decisions without any direct contact with the other firms of the game. The decisions of a period are communicated to the referee, often a computer, who calculates the results of the decisions, using the hidden functional relationships of the game. The results are communicated to the players who then make decisions for the next period of the game. In a negotiation game, the competing firms of the game enter into direct contact, as they have to negotiate deals or contracts with each other. This means that the negotiation game becomes even more lively and entertaining than the decision game, and also, what is more important, develops the negotiation skills of the participants.

The reinsurance game is played by a number of simulated insurance companies, each constituted by one player or a group of players. Each company starts the game with a certain net capital (surplus) and a portfolio of insurances. This portfolio is unchanged during the game, whereas the net capital changes

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[1] For references to the growing literature on the subject see the section on "Games—Industrial" in "Bibliography on the Use of Simulation in Management Analysis" by D. G. Malcolm. *Operations Research*, March-April 1960, pp. 169-177.

[2] J. S. McGuinness, "A Managerial Game for an Insurance Company," *Operations Research*, March-April 1960, pp. 196-209.

[3] The author, who will be grateful for communications on experiences at other institutions, is willing to furnish a set of data sheets for use in a game. His address is 10 Julius Thomsens Plads, Copenhagen V., Denmark.

[4] Another negotiation game is described by Green and Sisson, *Dynamic Management Decision Games*, Ch. 10, New York, 1959.

with the result of each period. In the game, it is the task of the companies both to reinsure their portfolios with other companies of the game in the way they think most suitable, and to obtain reinsurance business from other companies. The companies are free to enter any kind of reinsurance or retrocession agreement with each other. The number of companies should not be less than ten in order to establish sufficient possibilities for risk spreading. For practical reasons it is thought advisable to limit the number of companies to not more than twenty.

The Assumptions of the Game

In order to make any game playable, a number of simplifying assumptions have to be made. The loss results of a real insurance portfolio fluctuate from year to year for two very different reasons: First, there will be stochastic or chance deviations from the statistically expected loss figure and, second, this figure, the loss expectation, will change with the composition of the portfolio and with the various conditions that influence the occurrence of losses. In the model of the game, only stochastic deviations are considered. This means that the size and the composition of the portfolio are fixed, and so are the non-random loss influencing factors, and consequently, the loss result expectation. In practice, of course, the underwriter has to judge also the non-random factors from his knowledge of the line of insurance and the business of the company being considered. It is difficult for practical reasons to describe these circumstances in the game. Although these circumstances would influence reinsurance terms, it should be remembered that the primary purpose of reinsurance is to take care of random loss variations. The consideration of random variations only, therefore, may be an advantageous simplification. The implications of giving up the assumption of constant loss expectation will be mentioned briefly below.

Because of the loss expectation simplification, there is no reason to think of any definite line of insurance when the game is played. The line may be any one with fluctuating insurance amounts. In surplus reinsurance, big and small insurance amounts are reinsured in a different way. For practical reasons only three insurance amount sizes, $10,000, $50,000, and $500,000, are considered possible in the game. Furthermore, for the same reason, only six different loss sizes $500,000, $200,000, $100,000, $50,000, $25,000, and "small losses" are considered possible. These practical simplifications do not detract to any significant extent from the value of the game. When losses are considered for instance, the important factor is whether a loss is big, medium or small. The exact size is less important. In business practice, statistical tabulations of losses are divided into loss size groups.

The game is based upon a model of the portfolio of each company. An example of this model, which is constructed by the referee of the game, and is known by him only, appears as Form 1, the Data Sheet of the Company. The loss expectation shown in the upper part of this table, of course, is derived from the expected loss result distributions given in the lower part of the table. The annual premium income of the various companies differs in its distribution among insurance amounts and so does, consequently, the standard deviation of the expected loss amount distribution. In order to place the companies on a equal footing, as far as possible, the models are constructed in such a way that the expected underwriting profit of the direct business of each company (in the example: $10,650,000 — $10,191,000 = $459,000) is approximately proportional to the standard deviation of the annual loss amounts of the company.

Corresponding to the experience usu-

ally found in practice, the expected distributions of loss numbers according to loss size are constructed in such a way that there is a rather regular growth in the expected number of losses as loss size decreases. In our example the ratio between the loss numbers of two successive loss sizes is about 3, but the ratio varies from portfolio to portfolio in the game. The actual loss numbers in each loss size group of big losses are assumed to be Poisson-distributed around the expectation of the group. This will approximately be true, if losses occur independently of each other, and if the loss frequency is small. This will usually be the case in lines other than automobile insurance. From this assumption, annual loss numbers can easily be simulated, using random figures to enter the accumulated Poisson distribution of the expected loss number.[5] The loss amounts of "small losses" are assumed to be normally distributed and can be simulated by random figures when the loss expectation is supplemented with an assumption concerning the standard deviation.[6]

When the model of the loss result distribution of a company is constructed, the variance (the square of the standard deviation) of the annual loss amounts of

[5] Accumulated Poisson distributions are found in: E. C. Molina, *Poisson's Binomial Limit*, New York, D. Van Nostrand Co., 1942.

[6] Various collections of random normal deviates are published, e.g., *Tracts for Computers* XXV, Cambridge University Press.

Form 1. Data Sheet. Company No. 0.

	Insurance Amount			
	$10,000	*$50,000*	*$500,000*	*Total*
Annual premiums (in $000)	6,500	10,000	1,700	18,200
Loss expectation (in $000)	2,980	5,990	1,221	10,191

Loss allowance in premiums = Premiums − (expenses or 41.5 per cent of premiums) = $10,650,000

Simulated Loss Results of Years 1–20

Insurance amount	*$10,000*	*$50,000*			*$500,000*					
Loss size	*Small*	*$50,000*	*$25,000*	*Small*	*$500,000*	*$200,000*	*$100,000*	*$50,000*	*$25,000*	*Small*
Expectation *	*2980*	*13,1*	*31*	*4560*	*0,08*	*0,23*	*0,7*	*2,5*	*5,8*	*820*
Year 1	2968	10	31	4037	0	0	2	1	7	777
" 2	3075	13	25	4180	0	0	0	1	5	694
" 3	3028	16	29	4806	0	0	1	4	3	860
" 4	3144	18	38	4852	0	0	0	0	2	904
" 5	3117	14	32	4596	0	0	0	2	10	819
Year 6	3136	16	39	4940	0	0	1	1	11	868
" 7	3053	13	31	4572	0	0	1	1	5	727
" 8	3149	16	27	4748	0	0	4	1	6	803
" 9	3008	8	27	5034	0	0	0	3	7	885
" 10	2847	13	28	4873	0	0	0	1	4	884
" 11	2861	10	36	4776	0	0	1	2	9	961
" 12	3078	21	24	4533	0	0	3	1	3	697
" 13	2952	10	37	4730	0	0	0	1	3	1041
" 14	3065	17	36	4572	0	0	0	1	4	912
" 15	3133	10	37	4286	0	0	2	0	8	874
" 16	3020	13	38	5089	0	0	0	3	8	927
" 17	2843	15	23	4633	0	1	2	5	5	818
" 18	3057	15	29	4931	0	0	1	2	4	927
" 19	2861	12	31	4162	0	0	1	1	7	943
" 20	3074	11	38	4381	0	0	0	2	8	984

* Small losses in amount (in $000), other losses in numbers.

Form 2. Accounting Sheet. Company No. 0.

	Insurance Amount			
	$10,000	*$50,000*	*$500,000*	*Total*
Annual premiums (in $ 000)	6,500	10,000	1,700	18,200

Loss allowance in premiums = $18,200,000 − (expense loading or (41.5% × $18,200,000) = $10,650,000.)

Survey of Annual Results
(in $000)

Insurance Amount	*Loss Size*	*Year 1*	*Year 2*	*Year 3*	*Year 4*	*Year 5*	*Year 6*	*Year 7*	*Etc.*
$ 10,000	Total	2968	3075	3028	3144	3117	3136		
$ 50,000	$ 50,000	500	650	800	900	700	800		
	$ 25,000	775	625	725	950	800	975		
	Small	4037	4180	4806	4852	4596	4940		
	Total	5312	5455	6331	6702	6096	6715		
$ 500,000	$500,000	0	0	0	0	0	0		
	$200,000	0	0	0	0	0	0		
	$100,000	200	0	100	0	0	100		
	$ 50,000	50	50	200	0	100	50		
	$ 25,000	175	125	75	50	250	275		
	Small	777	694	860	904	819	868		
	Total	1202	869	1235	954	1169	1293		
Total losses, direct business		9482	9399	10594	10800	10382	11144		
Profit before reinsurance		1168	1251	56	−150	268	−494		
Net result of ceded business							−112		
Net result of accepted reins							500		
Net result of the year							−106		
Net capital, end of year						10000	9894		

the portfolio can easily be computed in this way: the numbers of losses in each loss size group are considered to be independent of each other. The variance of the total loss amount is therefore equal to the sum of the variances of the loss amounts of each group. Of these variances, the variance of the amount of small losses is simply equal to the square of the assumed standard deviation while, because of the Poisson distribution assumption, the variance of the amount of other losses is equal to the expected loss number multiplied by the square of the considered loss size.

Procedure of the Game

Before the game, the players are given the loss experience of their company for the five previous years. This information is given on Form 2, the Accounting Sheet, which is filled in by the players as the game goes on. Each company will have to set up other more detailed accounting sheets showing the results of its reinsurance operations, as Form 2 has space only for the net figures. The last line of Form 2 also informs the player about the net capital of his company when the game starts, after the end of year 5.

As already stated, the companies in the game are free to enter any kind of reinsurance agreement with each other. The most common forms, naturally, will be surplus and quota share treaties and excess of loss and excess of loss ratio contracts. Due to the insurance amount simplification of the game, surplus reinsurance will be similar to quota share reinsurance. If, say, a company decides to cede surpluses above a net retention of $40,000, this will mean that the company cedes a share of 20 per cent ($10,000) of all insurances of size $50,000, and a share of 92 per cent ($460,000) of all insurances of size $500,000.

In order to facilitate the game as far as possible, the players are supplied with a number of forms. Among these is a form (not illustrated) for the exchange of statistics and offers. Reinsurance contracts are entered in one of four forms. One of these, Form 3, for quota share reinsurance, and therefore, as shown by our example, also for surplus reinsurance, is shown. No attempt is made to cover all the terms of a reinsurance contract in the forms but space is provided for all necessary information concerning the operation of the treaty. Clauses on forwarding of deficits, inclusion of management allowances in the profit calculation, etc., may be indicated under "Other provisions". The treaty form also has space for statistics on treaty results. This, of course, is not in accordance with ordinary procedure, but it is useful in the game.

In order to illustrate the use of the forms, the figures corresponding to a net retention of $40,000 (cf. our example

Form 3. Quota Share Treaty. Contract No.....

Ceding Company: Company No. 0 Signature:.....

Reinsurer: Company No. .... Signature:.....

Shares: 20% of the $50,000—business of the ceding company
92% of the $500,000—business of the ceding company.

Commission: 25%

Profit Commission: 20%

Other provisions: Deficits of two previous years, if any, to be carried forward in the profit calculation.

Into force: Beginning of year...6... Out of force: End of year.....

Annual Treaty Results
(in $ 000)

	Year 6					
Premiums.....	3564					
Commission.....	891					
Losses.....	2533					
.....						
.....						
Gross profit.....	140					
Profit comm.....	28					
Net profit.....	112					
.....						
.....						

above) have been shown in the reproduction of the forms.

Treaty forms similar to Form 3, but adapted to the characteristics of the other forms of reinsurance, are also available for excess of loss contracts and excess of loss ratio (stop loss) contracts. Finally on a more general form, any kind of reinsurance agreement which the players know of, or can think up themselves, may be entered.

When the time which has been allowed the players for the conclusion of reinsurance agreements for the first period of the game, year 6, has passed, information on the loss result of the direct portfolios for this period is given to the players by the referee. For this purpose, Form 4 is

Form 4. Loss Results.

Company No. 0 Year: 6.

Ins. Amount	*Loss Size*	*Losses*	
		Number	*Amt. in $ 000*
$ 10,000	Small		3136
$ 50,000	$ 50,000	16	
	$ 25,000	39	
	Small		4940
$500,000	$500,000	0	
	$200,000	0	
	$100,000	1	
	$ 50,000	1	
	$ 25,000	11	
	Small		868

used. The needed copies of this form may be filled in by the referee before the game starts, as the decisions of the players do not influence the loss results. The referee has no computational work during the game. His work, the production of random loss results, is done before the game. This is in direct contrast to what is the case in all other known business games. The players themselves, on the basis of the contracts they have concluded, and the loss results communicated to them by the referee, do all the necessary calculations themselves. For this work they themselves set up the tables and sheets they think fitting. Form 5, a Settlement

Form 5. Settlement (in $000)

Year: 6 Contract No.....

Ceding Company: No. 0

Reinsurer: No.....

	Total Business	*Total Cession*	*Your Share*
Premiums		3564	
Commission		891	
Losses.....		2533	
.....			
.....			
Gross profit.....		140	
Profit commission.....		28	
Net profit to reinsurer..		112	

Note, of which a big number will be needed, is supplied by the referee.

As in reinsurance practice, the contracts of the game are concluded for an indefinite period until notice of termination is given by one of the parties. In the period between the communication of loss results for year 6 and year 7, the players, therefore, besides settling their accounts for year 6, are entitled to terminate agreements, alter terms, and conclude new agreements. Then loss results for year 7 are communicated, and accounts are settled and agreements negotiated anew on the basis of the new loss experience.

In this way, the game can go on as long as wanted. If possible, it should continue for a dozen periods, in order that the players will get a chance to learn from their experiences, and in order that the total result of each company will not be affected too much by the random deviations of the single years.

Applications and Experiences

At the Copenhagen School of Economics and Business Administration, the game was played for the first time in April

1960 at the end of a course in reinsurance. The model and the rules of the game had been explained, and the students had been allowed time for the analysis of their own first five year loss experiences before the game started. They had also had a chance to exchange statistics on this period, and to ask each other for offers concerning the terms of the agreements they would like to conclude. When the game started, therefore, one hour was enough for the conclusion of contracts for the first year. The second year was played through in forty-five minutes, and for successive periods, thirty minutes were allowed. The results, in form of the net capital of each company at the end of each period, were entered on a blackboard.

The experiences of the first game were highly satisfactory. The students not only enjoyed the thrill of the game, but said that the opportunity of doing reinsurance business themselves had made their understanding of the problems of reinsurance much more realistic than it had been.

In the academic year 1960-61, the game will be played simultaneously with the course. It will be started when the fundamentals have been taught in the class. Students will do the necessary negotiations outside of class hours, loss results only being communicated at the end of each class hour. Agreements will be entered in duplicate, one copy being given to the referee (the teacher), who will comment upon the development of the game as it goes on and draw upon the numerical examples of the game when the various treaty forms, the statistical analysis of data, and the problems of reinsurance terms are discussed in detail. It is hoped that both the teaching and the game will benefit from this interaction. The game has been played twice by members of the Insurance Society of Copenhagen and is also being considered as a tool in company training programmes.

Possible Modifications of Game Model

The game so far has been described in its simplest form. If wanted, it may, however, be modified in various ways in order to become more realistic. Whether complicating modifications are desirable or not, is a practical matter to be decided for each game. A few possible modifications shall be mentioned briefly.

It was assumed above that each player was free to reinsure as he liked and therefore also to choose freely the aim to be pursued in his reinsurance policy. A simple aim would be to survive the game with the biggest possible net capital at the end. It might, however, be included in the game, as a rule, that net result deviations for each company's own account should be reduced to a certain limit, and that the referee should fine a company penalty points, according to a progressive scale, if the net loss in a year passed a certain limit. Another more realistic form of punishment would be to reduce the annual premium income of the company that experienced such a loss. This, however, would make it necessary to change the model of the company during the game, and random loss results would have to be produced accordingly.

Whereas the change of model just mentioned was a function of the results of the game, an entirely different modification would be to construct the model of each company in such a way that premium income and loss result expectations changed in a way, not necessarily proportional, that was determined before the game. This assumption would make the game more realistic in the sense that players would have to give more weight to later loss results when revising their estimates of the loss result expectation of the portfolio, since these results would have been drawn from a population more similar to the population of the moment

of decision.[7] In real life, on the other hand, it might be that information other than premium income and aggregate loss results would be available for loss result estimation.

In reinsurance practice, considerations other than random deviations may enter into the decisions. Among these may be administrative costs of reinsurance, and liquidity and interest considerations. In the game, the cost element can be included through a rule according to which the parties shall debit themselves administrative costs according to a scale which may include a certain percentage of ceded premiums in the case of surplus reinsurance. In other cases it may be reasonable simply to fix the administrative costs of each party as a flat amount per treaty. Interest considerations might be included through a rule according to which each company, for each period, had to decide how much capital the company wanted to keep in liquid form. The company would have to debit itself a corresponding loss of interest. If the liquid capital were too small to cover a net loss, the company would have to borrow from the referee who would act as a bank, according to rules laid down beforehand.

Conclusion

Although a reinsurance game probably will be most valuable for educational purposes, it should be mentioned in conclusion, that it can be used also in research work, since it shares with other business games the advantage that one can carry out laboratory experiments under controlled conditions. This privilege was previously reserved for the natural sciences. If the purpose is to study behavior and reactions in the reinsurance market, the game should be played by experienced insurance people. It may also be used for experiments concerning new forms of reinsurance.

[7] For experiments on decision-making in a similar choice situation see R. J. Schreiber, "Estimates of Expected Value as a Function of Distribution Parameters," *Journal of Experimental Psychology* 53, 1957, 218-220.